

# Investigation the Accuracy of Different Global Geo-Potential Models Data over Egypt

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**Abstract**— Geoid Undulation value can be calculated by several GGMs and due to the huge area of Egypt, each GGM calculates a different undulation value of the same point, therefore the accuracy for the calculated geoid undulation values will affecting directly on the accuracy of the converting process between Geodetic height and Orthometric height. The main objective of this paper is investigating the accuracy of different Global Geo-Potential Models data which recently used over Egypt. Eight GGMs were selected to be tested in this study under different standards. 346 stations were regularly distributed over Egypt territories as network; these are bounded by latitudes [22° N, 31° N] and by longitudes [26° E, 36° E]. The methodology of this paper had been based on utilizing the least-squares theory, the Coefficient of Variation (C.V) ratio were calculated to compare the results for each selected GGM. Generally, the maximum (C.V) ratio reach 18.418 % of EIGEN-CG01C and the minimum ratio reach 16.829 % of EIGEN-GL04C. Finally, this paper recommends using six GGMs over Egypt territories, this is based on insignificant variations between these six GGMs after using, testing and analyzing the all geoid undulation values for the 346 stations.

**Index Terms**— investigate, accuracy, Coefficient of Variation, Geoid, Undulation, GGM, Ellipsoidal heights, Orthometric heights, Egypt.

## 1 INTRODUCTION

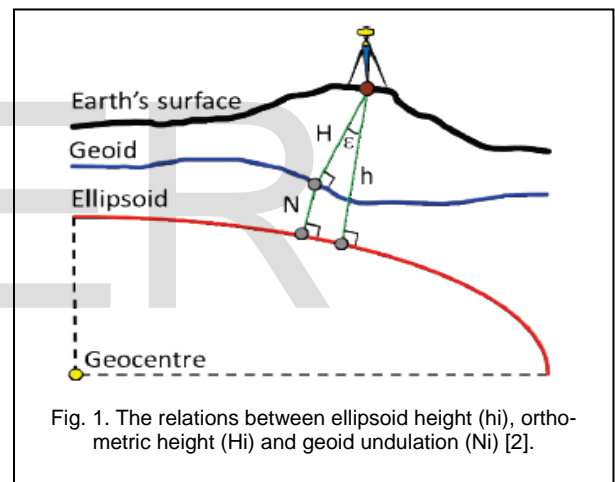
Specific three-dimensional (3D) location on ground or map called "Point Coordinate". 3D Point Coordinate is [Longitude ( $\lambda$ ), Latitude ( $\phi$ ) and Height ( $h/H$ )]. The vertical component ( $h/H$ ) is the main concern of this paper.

The vertical position has two main factors according to the used reference surface; **Ellipsoidal datum** (is based on a geometric model) and the **Equipotential surface (the geoid)** (is the surface of constant gravity potential) [1]. Therefore, the vertical component has two different heights values **Ellipsoidal heights/Geodetic heights ( $h_i$ )** and **Orthometric heights ( $H_i$ )**. While ( $H_i$ ) can be measured by traditional field survey techniques (Spirit Levelling or Total-Station), ( $h_i$ ) can be measured by Global Positioning System (GPS).

In order to utilize the advantages of space-based technique, in last 20 years the GPS data ( $h_i$ ) became a widespread in Egypt which referred usually to the World Geodetic System 1984 (WGS84). On the other side, ( $H_i$ ) is needed still for most of civil engineering projects (engineering applications and mapping processes). Therefore the transforming is needed between the both heights ( $h_i$ ) and ( $H_i$ ).

Equation (1) presents the mathematical relationship between ( $h_i$ ) and ( $H_i$ ) as shown in Fig. (1), where ( $N_i$ ) called the **Geoid Undulation/Geoidal Height**. Therefore, ( $N_i$ ) value for any point (i) can be used to convert ( $H_i$ ) value and ( $h_i$ ) value in both directions.

$$N_i = h_i - H_i \quad \dots\dots(1)$$



**Global Geopotential Models (GGMs)** is being represented by the spherical harmonic coefficient which defines the potential of gravitational in the spectral domain [3]. Several GGMs are used to calculate ( $N_i$ ) value at any point (i). Egypt has a large area which up to 1000000 km<sup>2</sup>, each GGM calculates a different ( $N_i$ ) value of the same point [4]. The accuracy for the calculated ( $N_i$ ) by each GGM will affecting directly on the accuracy of the converting process between ( $h_i$ ) and ( $H_i$ ). This paper aims to investigate the accuracy of different Global Geo-Potential Models Data which recently used over Egypt.

## 2 RELATED WORK

A different data were collected from different satellite missions GRACE (Gravity Recovery And Climate Experiment), GOCE (Global Ocean Circulation Experiment), LAGEOS (Laser GEodynamic Satellite) and CHAMP (CHALLENGING Minisatellite Payload) to strengthen different GGMs results. Anyway, the GGMs data can be classified from satellite only models or combined model between different kinds of data [3].

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According to the importance of Egypt's area and location importance of Egypt many trials were conducted by several researchers. The main target of all these trials is enhancement the GGMs results to be as the best fit to reality over Egypt territories [5]. Other Sub-goals were presented by different researchers using GGMs data over Egypt such as quantify the precision [6], development the models ([7],[8],[9],[10],[11]), study the relation between different models [4], Deriving a trusted Geoid Undulation Network (N) by Merging Data of Different models [12], investigate the models accuracy [13] and evaluate the performance of models ([14],[15]). Also, the researchers met a challenge task to select a limited number of GGMs to study since there are about 160 models known GGMs in world [13]. The main standards were taken into considered to choose some of GGMs for each study are the accurate, availability, the format of dataset, variation in the spherical harmonic coefficient, the most common used in study area recently and the variation in the issue year [12].

For Egypt the researchers tested about thirty GGMs and they recommended 14 GGMs in different studies, as shown in table (1), as a most models used over Egypt recently in last 2 decades.

TABLE 1

14 GGMs WERE RECOMMENDED TO COVER EGYPT TERRITORIES.

Model	Year	Degree	Authors
GGM02C	2004	200	10 GGMs [5] 8 GGMs ([4],[12])
EIGEN GRACE-02S	2004	150	
EGM 2008	2008	360	
EIGEN-CG01C	2004	360	
GGM03C	2009	360	
EIGEN-CG03C	2005	360	
EIGEN-GL04C	2006	360	
EIGEN-05C	2008	360	
EGM96	1996	360	
EGM 2008	2008	2190	
DGM-1S	2012	250	7 GGMs [6]
GO_CONS_GCF_2_TIM_R5	2014	280	
GO_CONS_GCF_2_DIR_R5	2014	300	
EIGEN-6C4	2014	2190	

Where; S = Satellite Tracking Data, G = Terrestrial Gravity Data, A = Altimetry Data and (CHAMP, GOCE, GRACE, and LAGEOS) are gravity satellite missions.

This paper aim is to investigate the accuracy of different Global Geo-Potential Models (GGMs) Data.

### 3 STUDY AREA AND DATA OVER EGYPT

According to the standards were taken into consideration which was mentioned before in the related work section; eight GGMs are selected to be tested in this paper, as shoown in table (1), from the recommended GGMs by other authors as the most common used GGMS in in the last 25 years over Egypt and also the available GGMs with variation in the spherical harmonic coefficient and the issue year. The eight

selected GGMs are EGM96, EIGEN-CG01C, GGM03C, EIGEN-CG03C, EIGEN-GL04C, EIGEN-05C, EGM 2008 (360) and EGM 2008 (2190).

A regularly 346 stations were distributed to cover whole area of Egypt territories as network. This network is bounded by latitudes ( $\phi$ ) [22° N, 31° N] and by longitudes ( $\lambda$ ) [26° E, 36° E]. The network's interval is 0.5° in the both directions ( $\phi$ ,  $\lambda$ ) with distance about 50 Km between each two neighbor stations as shown in figure (2).

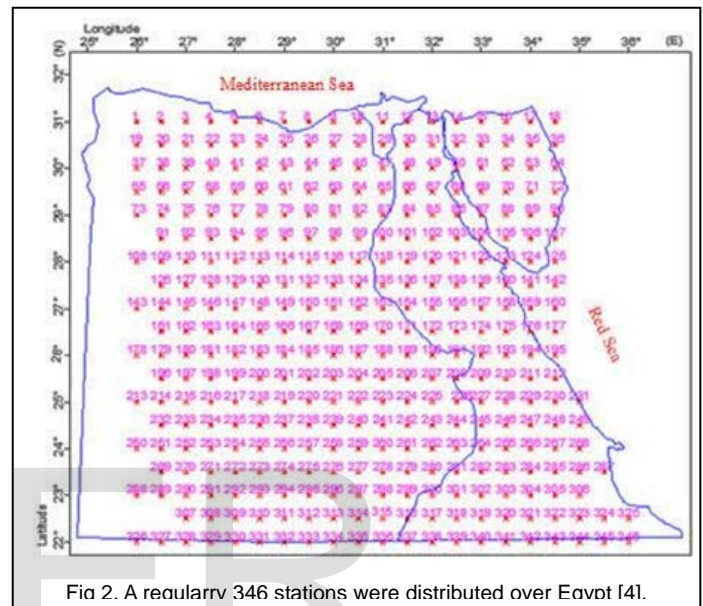


Fig 2. A regular 346 stations were distributed over Egypt [4].

Geoid undulation ( $N_i$ ) values were calculated for 346 stations by each one of eight different GGMs using equations 1. In order to avoid the prolongation in this paper, table (2) presented sample of the calculated Geoid undulation ( $N_i$ ) values for 21 stations only as example, which are located on ( $\phi = 22^\circ$  North) and the stations ID as shown in figure (2). On the other hand, the final conclusion of this study will depend using, testing and analyzing the all results of the 346 stations for each GGM of the eight selected GGMs.

$$N = \left( \frac{GM}{r\gamma} \right) \sum_{n=2}^{n=\max} \left( \frac{a}{r} \right)^n - \sum_{m=0}^n [(\bar{C}_{nm} \cos m\lambda) + (\bar{S}_{nm} \sin m\lambda)] \bar{P}_{nm} \sin \phi \quad \dots(1)$$

Where  $G$  is the Newtonian gravitational Constant,  $m$  is the mass of the earth,  $n$  is the degree of the geo-potential model,  $(\bar{C}_{nm}, \bar{S}_{nm})$  are the fully normalized geo-potential coefficients of degree and order  $(n, m)$ ,  $\bar{P}_{nm}$  is the fully normalized associated Legendre function of degree and order  $(n, m)$ ,  $a$  is the semi-major axis,  $\gamma$  is the normal gravity on the reference ellipsoid,  $r$  is the radial distance from Earth's mass center and  $(\phi, \lambda)$  are the geocentric latitude and longitude of each station.

TABLE 2  
SAMPLE FOR 21 STATIONS ONLY OF CALCULATED ( $N_i$ ) VALUES BY EIGHT GGMS AS AN EXAMPLE.

Station ID ( $i$ )	$k$		1	2	3	4	5	6	7	8
	Lat. (Deg.)	Long. (Deg.)	EGM96	EIGEN- CG01C	GGM03C	EIGEN- CG03C	EIGEN- GL04C	EIGEN-5C	EGM 2008 (360)	EGM 2008 (2190)
326	22	26	15.85	15.67	16.69	16.50	16.86	16.61	16.95	16.58
327	22	26.5	15.33	15.09	15.86	16.03	15.52	15.73	16.07	15.80
328	22	27	14.22	13.74	14.34	14.58	13.84	14.22	14.70	14.39
329	22	27.5	13.78	12.97	13.57	13.62	13.34	13.67	13.96	13.62
330	22	28	13.45	12.46	13.16	13.00	13.09	13.06	13.34	13.09
331	22	28.5	12.81	11.67	12.63	12.27	12.63	12.44	12.59	12.35
332	22	29	12.40	11.17	12.23	11.96	12.17	12.13	12.22	11.83
333	22	29.5	11.89	10.87	11.79	11.79	11.68	11.81	11.79	11.47
334	22	30	11.36	10.60	11.33	11.46	11.17	11.50	11.31	11.08
335	22	30.5	10.78	10.47	10.96	11.11	10.92	11.05	11.00	10.67
336	22	31	10.35	10.46	10.83	10.90	10.96	10.83	10.82	10.52
337	22	31.5	10.12	10.43	10.87	10.87	11.07	10.96	10.72	10.44
338	22	32	9.66	9.93	10.69	10.60	10.68	10.81	10.38	10.07
339	22	32.5	9.56	9.51	10.60	10.50	10.29	10.68	10.18	9.86
340	22	33	9.62	9.26	10.14	10.44	10.01	10.22	10.03	9.59
341	22	33.5	9.75	8.85	9.66	9.96	9.78	9.55	9.80	9.59
342	22	34	10.73	9.47	10.26	10.31	10.53	10.23	10.22	9.91
343	22	34.5	11.24	9.71	10.54	10.30	10.72	10.72	10.37	10.21
344	22	35	10.98	9.36	10.14	9.89	10.41	10.58	10.42	9.93
345	22	35.5	9.94	8.53	9.37	9.24	9.47	9.51	9.63	9.85
346	22	36	8.87	7.61	8.56	8.57	8.43	8.25	8.46	7.87

$$\left[ \sum (v^2)_k = (v_1^2)_k + (v_2^2)_k + \dots + (v_i^2)_k \right].$$

### 3 METHODOLOGY AND TESTS

Least-squares method was utilized to methodology for this study as follows;

- I) Each one of GGMS ( $k = 1 \rightarrow 8$ ) was used to calculate undulation values ( $N_i$ ), where ( $i = 1 \rightarrow 346$ ) stations), the stations were distributed regularly over Egypt with interval  $0.5^\circ$  in both directions [latitudes ( $\phi$ ) and longitudes ( $\lambda$ )].
- II) The mean (the Most Probable Value) was,  $[(\bar{N}_i)_k = \sum (N_i)_k / \sum (i)]$ , calculated For Each GGM ( $k$ ). Where  $\sum (i) = 346$
- III) The residual Error ( $v$ ) was calculated at each Station ( $i$ )  $[(v_i)_k = (N_i)_k - (\bar{N}_i)_k]$ .
- IV) The square of the residual Error  $\sum (v^2)_k$  was calculated for each GGM ( $k$ )

- V) The Standard Deviation ( $SD$ ) $_k$  was calculated for each GGM ( $k$ )  $[(SD)_k = \sqrt{\sum (v^2)_k / (i-1)}]$ .

- VI) The coefficient of variation/variance ratio ( $C.V$ ) $_k$  was calculated for each GGM ( $k$ )

$$[(C.V)_k = \{(SD)_k / (\bar{N}_i)_k\} \times 100]$$

- VII) The coefficient of variation ratio is used to compare the variability between the eight GGMS ( $k = 1 \rightarrow 8$ ) relative to its mean.
- VIII) In other words, that GGM which has a higher ratio

percentage of ( $C.V$ ) $_k$ , it will be more variation and consequently its accuracy will be lower than the other GGMS. That is due to the all input-data ( $\phi, \lambda$ ) are constant in this study for the 346 stations, therefore any change of the coefficient of variation values will leads directly to the accuracy of GGM itself.

TABLE 3  
THE RESULTS OF DIFFERENT PARAMETERS WERE CALCULATED FOR EACH GGM OF EIGHT GGMs.

$k$	1	2	3	4	5	6	7	8
Symbol	EGM96	EIGEN- CG01C	GGM03C	EIGEN- CG03C	EIGEN- GL04C	EIGEN-5C	EGM 2008 (360)	EGM 2008 (2190)
$\sum (N_i)_k$	4869.601	4791.161	4973.54	4970.175	4970.409	4974.151	4971.712	4856.312
$(\bar{N}_i)_k$	14.074	13.847	14.374	14.365	14.365	14.376	14.369	14.036
$\sum (v^2)_k$	1967.47	2244.009	2023.95	2055.473	2016.305	2037.569	2026.507	1991.923
$(SD)_k$	2.388	2.550	2.422	2.441	2.418	2.430	2.424	2.403
$(C.V)_k$	0.170	0.184	0.169	0.170	0.168	0.169	0.169	0.171
$(C.V)_k \%$	16.968	18.418	16.850	16.992	16.829	16.905	16.867	17.120
Re-sorting (Smallest to Largest)	5	8	2	6	1	4	3	7

Table (3) presents the final results of different parameters were calculated respectively for each GGM of the different Eight GGMs. Also, the coefficient of variation ratio is shown in figure (3) for the eight GGMs.

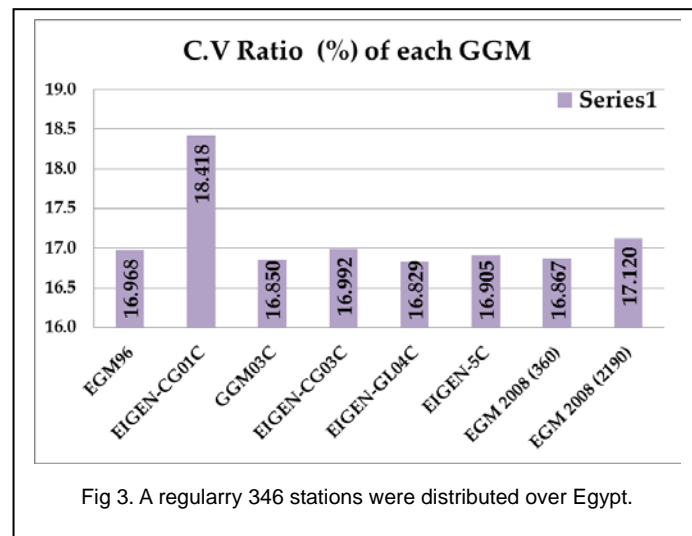


Fig 3. A regular 346 stations were distributed over Egypt.

According to Table (3) and figure (3), the coefficient of variation ratio  $(C.V)_k$  for the eight GGMs can be sorting from smallest to largest ratio. The GGM which has a smallest ratio, it is will have a less variation.

As shown in table (3), EIGEN-GL04C(360)[2006], GGM03C(360)[2009], EGM2008(360)[2008], EIGEN-5C(360)[2008], EGM96(360)[1996], EIGEN-CG03C(360)[2005], EGM2008(2190)[2008] and EIGEN-CG01C(360)[2004] were re-sorted respectively according to the accuracy between each other's relatively.

While many previous studies classified EGM 2008-2190(2008) as a best fit to reality and most common GGM used in Egypt recently, but in this paper showed for EGM2008(2190)[2008] a higher variation more than the other GGMs (where, its  $C.V$  ratio is 17.120 % and its rank number seven among eight GGMs).

Anyway, this paper recommends six (from 1 to 6 of resorting cell) GGMs results to use over Egypt territories as priority. That is based on insignificant variations between these six GGMs, since the calculated variation difference between the maximum (16.992 % of GGM03C(360)[2009]) and the minimum (16.829 % of EIGEN-GL04C(360)[2006]) coefficient of variation ratio  $(C.V)_k$  of these six GGMs is 0.163 % only.

## 5 CONCLUSIONS

$(N_i)$  value at any point (i) can be calculated by several GGMs and due to the huge area of Egypt (up to 1000000 km<sup>2</sup>), each GGM calculates a different  $(N_i)$  value of the same point [4], therefore the accuracy for the calculated values  $(N_i)$  by each GGM will affecting directly on the accuracy of the converting process between  $(h_i)$  and  $(H_i)$ . The main objective of this paper



is investigating the accuracy of different Global Geo-Potential Models Data which recently used over Egypt.

Eight GGMs were selected to be tested in this study. several main standards were taken into considered to select these eight GGMs from 160 known GGMs over world such as the recommended GGMs over Egypt by other studies, the accuracy, availability, the format of dataset, variation in the spherical harmonic coefficient, the most common used in study area recently and the variation in the issue year. The eight selected GGMs are EGM96, EIGEN-CG01C, GGM03C, EIGEN-CG03C, EIGEN-GL04C, EIGEN-05C, EGM 2008 (360) and EGM 2008 (2190).

346 stations were regularly distributed over Egypt territories as network; these are bounded by latitudes ( $\phi$ ) [22° N, 31° N] and by longitudes ( $\lambda$ ) [26° E, 36° E] with interval 0.5° in the both directions ( $\phi$ ,  $\lambda$ ). The final conclusion is depending on use, test and analysis the all geoid undulation ( $N_i$ ) values for the 346 stations, which are calculated by each GGM of the eight selected GGMs.

The methodology of this paper had been based on utilizing the least-squares theory. Different parameters such as (Most Probable Value, Residual Errors square, Standard Deviation and The Coefficient of Variation ratio), were calculated respectively for each selected GGM. The eight coefficients of variation ratio were used to compare the variability between different GGMs relative to its mean. Since the all input-data ( $\phi$ ,  $\lambda$ ) are constant for the 346 stations, therefore any change of the coefficient of variation values will leads directly to the accuracy of GGM itself.

Then the eight GGMs were re-sorted based on the coefficient of variation ratio ( $C.V$ )<sub>k</sub> form smallest to largest value. The final re-sorting for the eight GGMs is mentioned respectively in table (3) as following; EIGEN-GL04C(360)[2006], GGM03C(360)[2009], EGM2008(360)[2008], EIGEN-5C(360)[2008], EGM96(360)[1996], EIGEN-CG03C(360)[2005], EGM2008(2190)[2008] and EIGEN-CG01C(360)[2004]. The calculated coefficient of variation ratio ( $C.V$ )<sub>k</sub> is presented the maximum ratio reach 18.418 % of EIGEN-CG01C and the minimum ratio reach 16.829 % of EIGEN-GL04C.

Finally, this paper recommends using the first six GGMs (from 1 to 6 of resorting cell) over Egypt territories as priority. That is based on insignificant variations between these six GGMs, since the calculated variation difference between the maximum value (reach 16.992 % of GGM03C(360)[2009]) and the minimum value (reach 16.829 % of EIGEN-GL04C(360)[2006]) for coefficient of variation ratio ( $C.V$ )<sub>k</sub> is 0.163 % only of these six GGMs.

The future recommendation of this paper is to go ahead more deeply by different statistical methods, by the field survey and different mathematical ways to enhancement GGMs results until the best fit to reality over Egypt is achieved.

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